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Extracting Meteorological Data from Projectile Trajectory

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The atmospheric conditions are found by knowing only the projectile's flight trajectory and its aerodynamic coefficients together with the initial atmospheric conditions on the ground. The test trajectories were generated as solutions of the modified point mass equations of motion. The correct atmospheric conditions for the generated flight trajectory are obtained from data collected during a weather balloon flight. A nonlinear least squares method then was used to fit the modified point mass equations to the test trajectory by varying the meteorological parameters. Density, temperature, and wind profiles agreed well.

Nomenclature

A = $[(\rho S\ell/2m)C_D](V/\ell)$, 1/s
AZ = azimuth of the 1 axis, measured clockwise from north, rad
a = $A/(V/\ell)$
B = $k_a^{-2}(\rho S\ell/2m)(V/\ell)^2$, 1/s^2
$\boldsymbol{C}$ = Coriolis acceleration, $-2\boldsymbol{\omega}\times\boldsymbol{U}$, 1/s^2
C_D = drag coefficient, where |drag force| equals $(\rho V^2 S/2)C_D$
C_{L_α} = gradient of lift force coefficient, where |lift force| equals $\pm(\rho V^2 S/2)|\alpha_e|C_{L_\alpha}$
C_{ℓ_p} = roll damping moment coefficient, where |roll damping moment| equals $\pm(\rho V^2 S\ell/2)(\dot\phi\ell/V)C_{\ell_p}$
C_{M_α} = gradient of pitching moment coefficient, where |static moment| equals $\pm(\rho V^2 S\ell/2)|\alpha_e|C_{M_\alpha}$
$C_{N_{p\alpha}}$ = gradient of Magnus force coefficient, where |Magnus force| equals $\pm(\rho V^2 S/2)(\dot\phi\ell/V)|\alpha_e|C_{N_{p\alpha}}$
C_1, C_2 = parameters to fit the wind, m/s
C_3 = parameter to fit the temperature gradient, K/km
D = $h_a(\boldsymbol{G}\cdot\boldsymbol{V})/(1+h_a)V^2$, 1/s
E = gun elevation, rad
E_ρ = relative error in the fitted air density
$\boldsymbol{G}$ = gravity plus Coriolis acceleration, $\boldsymbol{g}+\boldsymbol{C}$, m/s^2
$\boldsymbol{G}_A$ = $1/(1+h_a)\{\boldsymbol{G}+[h_L(\boldsymbol{G}\times\boldsymbol{V})/(1-h_M)V]\}$, m/s^2
g = $|\boldsymbol{g}|$, m/s^2
$\boldsymbol{g}$ = gravity acceleration, m/s^2
g_0 = $|\boldsymbol{g}|$ at sea level; U.S. Standard Atmosphere (STAT) value at 45°N latitude is 9.80665 m/s^2
H = geopotential altitude, m
H_A = value of H at the starting time of a fitting interval, m
H_B = value of H at the bottom of a STAT altitude zone, m
h_a = $[h_L^2/(1-h_M)]-h_M$
h_L = $k_a^2(C_{L_\alpha}/C_{M_\alpha})(\dot\phi\ell/V)$
h_M = $k_a^2(C_{N_{p\alpha}}/C_{M_\alpha})(\dot\phi\ell/V)^2$
I_x = axial moment of inertia, kgm^2
J = number of significant digits in the fitted air density
k_a^2 = $I_x/m\ell^2$
L = latitude at the launch point; for southern hemisphere firings, replace L by $-L$, rad
ℓ = reference length equal to diameter equal to caliber, m
$\mathcal{M}$ = molecular weight, kg/mol
m = projectile mass, kg
Q = constant in the STAT air density formula, $(g_0\mathcal{M})/\mathcal{R}$, K/km
R = effective radius of the Earth, 6,356,766 m
$\mathcal{R}$ = universal gas constant, J/(mol K)
S = reference area, $\pi\ell^2/4$, m^2
T = temperature, K
T' = temperature gradient $\mathrm{d}T/\mathrm{d}H$, K/km
T_A = temperature at the starting time of a fitting interval, K
T_B = STAT temperature at altitude H_B, K
t = time, s
$\bar t$ = nondimensionalized time, $U_{1b}(t-t_b)/\ell$
$\boldsymbol{U}$ = $\dot{\boldsymbol{X}}$, X projectile velocity with respect to the Earth, m/s
U_i = flat-Earth system components of $\boldsymbol{U}$, m/s
V = $|\boldsymbol{V}|$, m/s
$\boldsymbol{V}$ = projectile velocity with respect to air, $\boldsymbol{U}-\boldsymbol{W}$, m/s
V_s = speed of sound, m/s
$\boldsymbol{W}$ = wind velocity with respect to the Earth, m/s
W_i = flat Earth system components of $\boldsymbol{W}$, m/s
$\boldsymbol{X}$ = projectile position with respect to the Earth, m
X_i = flat Earth components of $\boldsymbol{X}$, right handed, m
X_1 = downrange
X_2 = height above sea level
X_3 = lateral
x = nondimensional position variable, $(X_1-X_{1b})/\ell$
$\boldsymbol{\alpha}_e$ = yaw of repose, rad
β = mass fraction of water vapor in the air
ρ = air density, kg/m^3
ρ_A = air density at the start of a fitting interval, kg/m^3
ρ_B = STAT air density at H_B, kg/m^3
$\dot\phi$ = axial spin rate, rad/s
$\boldsymbol{\omega}$ = angular velocity of the Earth, rad/s
$(\dot{\ })$ = $\mathrm{d}()/\mathrm{d}t$

Introduction

A BETTER knowledge of atmospheric conditions could improve the accuracy of artillery. Currently, weather balloons are used to gather the temperature, pressure, and wind velocity as a function of height above the Earth's surface. The data are presented in a formalized meteorological table format. As the weather changes, the information becomes stale and contributes significantly to the artillery round's error budget. Ideally, the atmospheric conditions need to be known immediately before a round is fired.[1]

Because the trajectory of a projectile depends on the atmospheric conditions, among other things, the problem can be turned around to determine the atmospheric conditions by knowing the trajectory of a fired projectile. More specifically, a nonlinear least squares method can be used with the trajectory data, if the equations of motion are assumed and the projectile's aerodynamic coefficients known,[2] to yield a best fit to the density, temperature, and wind velocity as a function of altitude. Thus, the atmospheric conditions would be determined soon after the first shot was fired, with the elapsed time for the extraction of the meteorological results depending on the computer's speed and the efficiency of the solution algorithm. The atmospheric quantities could subsequently be used with the artillery piece swung to a new azimuth.

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As a first step, Cooper and Bradley[3] posed a problem that did not use atmospheric data from a balloon flight; to use complicated experimental data would pose a more difficult problem. Their observed trajectory was calculated from the modified point mass (MPM) equations of motion[4,5] combined with the 1966 U.S. Standard Atmosphere[6] (STAT) and linear wind profiles. The STAT assumes a linear temperature with altitude and a simple relationship between the temperature and density. To facilitate fitting the manufactured trajectory data, Bradley[5] revised the MPM equations so that current fitting techniques could be adopted to iterate the air temperature gradient with respect to height and wind velocity as fittable parameters. FINLIE,[7] a nonlinear curve-fitting method, then fitted the revised MPM equations to the generated data by adjusting these parameter values. When the fitted values for trajectory, air density, air temperature, and wind profiles were compared with the generated corresponding values, the agreement was found to be excellent.

The work described in this report follows and expands that of Cooper and Bradley.[3] Instead of using an idealized atmospheric model with the wind velocity being linear with height above the ground, the atmospheric data from a weather balloon were used as input to generate the test trajectory. The test trajectory was then fitted over segments to obtain approximations to the wind velocities, density, and temperature. Error studies were also performed, with white noise being introduced onto the trajectory values to simulate the uncertainty in trajectory measured by radar or the global positioning satellite system (GPS) techniques.

Trajectory Equations and Coordinate System

The construction of firing tables depends significantly on the use of the MPM trajectory model. More complex models are used only for special cases, and the complete data set to construct these more complex models is not commonly available. The six-degree-of-freedom equations have time derivatives that do not appear in an explicit factored form. FINLIE, however, uses only equations with derivatives that are factored. Thus, the MPM equations will be used here in a different but almost equivalent form[4] as used elsewhere.[3]

Bradley's[5] formulation for the MPM model can be rewritten in the form

$$\dot{\boldsymbol{U}} = (D - A)\boldsymbol{V} + \boldsymbol{G}_A \tag{1}$$

The axial spin $\dot{\phi}$ is obtained as the solution of a simplified roll equation,

$$\ddot{\phi} = -BC_{\ell_p}(\dot{\phi}\ell/V) \tag{2}$$

The aerodynamic coefficient gradients $C_{L\alpha}$, $C_{M\alpha}$, $C_{Np\alpha}$, and C_{ℓ_p} in Eqs. (1) and (2) are tabulated as functions of Mach number. The drag coefficient C_D depends on both Mach number and the yaw of repose α_e. In particular, C_D is assumed to have the form

$$C_D = C_{D0} + C_{D2}|\boldsymbol{\alpha}_e|^2 \tag{3}$$

in which C_{D0} is a function of Mach number and C_{D2} is a constant.

The yaw of repose results from the curvature of the trajectory caused by the Earth's gravitational force and can be computed by

$$\boldsymbol{\alpha}_e = \frac{\dot{\phi}\boldsymbol{G}_A \times \boldsymbol{V}}{BV^2C_{M\alpha}} \tag{4}$$

A convenient coordinate system is needed for describing the motion of a projectile along its trajectory. Following Cooper and Bradley's[3] treatment, it is assumed that the launch point is at sea level with the launch point set at the origin. A right-handed Cartesian system is then defined as follows: the 1 and 3 axes form a plane tangent to the Earth at the origin; the 2 axis is perpendicular to this plane, positive upward, and the 1 axis is chosen so that the velocity $\boldsymbol{U}$ at time zero is in the 1–2 plane. The initial velocity is given by

$$U_0 = |\boldsymbol{U}_0|(\cos E, \sin E, 0) \tag{5}$$

Because the trajectory (in most cases) lies nearly in the 1–2 plane, X_3 is usually much smaller in magnitude than X_1 or X_2.

Equation (1) can be written in component form as

$$\dot{U}_1 = (D - A)V_1 + h\left[G_1 + \frac{h_L(G_2V_3 - G_3V_2)}{(1 - h_M)V}\right] \tag{6}$$

$$\dot{U}_2 = (D - A)V_2 + h\left[G_2 + \frac{h_L(G_3V_1 - G_1V_3)}{(1 - h_M)V}\right] \tag{7}$$

$$\dot{U}_3 = (D - A)V_3 + h\left[G_3 + \frac{h_L(G_1V_2 - G_2V_1)}{(1 - h_M)V}\right] \tag{8}$$

in which $h = 1/(1 + h_a)$. The influence equations include the yaw of repose but were not included in the former study.[3]

Atmospheric Data and Modeling of Atmosphere for Fitting Data

The atmospheric conditions supplied to the fitting procedure were obtained from a weather balloon. The atmospheric model, which is assumed to fit the data, was selected from some chosen models to best approximate the measured conditions. More complex models take more computer time without yielding a better fit to the data. The latitude L for the simulated test was 39.15° north, the azimuth AZ was 21.915°, and g_0 was 9.80665 m/s^2.

Atmosphere Used in the Generating Equations

The calculated test trajectories used balloon data taken at Aberdeen Proving Ground on March 30, 1992, in the early afternoon. The data consisted of pressure, temperature, and wind velocity in the horizontal plane taken at very small intervals of altitude initially and increasing to intervals of 400 m above altitudes of 3000 m. The wind velocity data are shown in a later section when comparisons are to be made with fitted values.

The temperature as a function of altitude is given in Fig. 1. Shown for comparison is the temperature curve obtained from the STAT.[6] The STAT curve can be considered as an approximation of the average temperature in temperate climates in North America. Although the recorded temperature varies no more than a few degrees from the STAT values, the temperature profile can vary markedly according to location and time of the year. The winter temperature profile in Siberia would have an initial positive gradient with respect to altitude, whereas the average temperature profile at the latitudes in North America would have a temperature gradient of opposite sign.

Atmosphere Model Used with Fitting Equations

Cooper and Bradley[3] fitted the horizontal components of the wind, as well as the density and temperature, in one segment with a linear curve. Not surprisingly, they obtained a good fit with the imposed linear wind velocities. For the present work, the quantities were fitted within a few time segments with the data equally weighted. A linear relationship for the wind velocities was first tried within each segment, but a constant value for the wind velocities throughout each

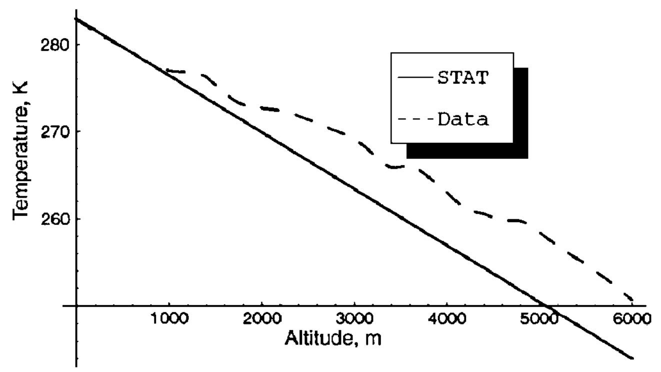

Fig. 1 Temperature data compared to standard atmosphere curve temperature.

time segment gave a better fit. More specifically, for each segment along the trajectory, the wind velocity is assumed as

$$W_1 = C_1 \tag{9}$$

$$W_2 = 0 \tag{10}$$

$$W_3 = C_2 \tag{11}$$

The rate of change of temperature with respect to altitude H is the third parameter to be fitted, $C_3 = T'$. Cooper and Bradley[3] assumed a temperature dependence (Kelvin) on the height of the form

$$T = T_B + T'(H - H_B) \tag{12}$$

in which the gradient of the temperature T' is the negative of the lapse rate. For those zones in which T' is not zero, the STAT air density is given by

$$\rho/\rho_B = (T/T_B)^{-1-Q/T'} \tag{13}$$

The value in Kelvin per kilometer of Q for dry air is

$$Q_d = 34.16319474 \tag{14}$$

For moist air, the value of Q can be approximated as

$$Q_m = \frac{Q_d}{1 + 0.61\beta} \tag{15}$$

The value of β can be as large as 0.04, but it is assumed that $\beta = 0$ for this analysis.

The current model fits the lapse rate and wind velocities on altitude zones assuming that the density varied as Eq. (13). The subscript values at the bottom of the corresponding altitude zones are denoted as A. As before, the first time interval values of T_A, H_A, and ρ_A (obtained by measurements at the launch site) are required inputs to the fitting process; thereafter, the closing values for the kth interval become the starting values for the $(k+1)$th interval.

Characteristics of the Test Projectile

Trajectories were generated for a M107 projectile, which has been well studied, and all of the coefficients needed for use with the MPM equations are well known.[2] This projectile has the following physical properties: ℓ (diameter) = 155 mm, m (mass) = 43.09 kg, and $I_x = 0.1461$ kgm^2.

Of the six aerodynamic coefficients and gradients of coefficients involved in the equations of motion, C_{D0}, $C_{L\alpha}$, $C_{M\alpha}$, C_{ℓ_p}, C_{D2}, and $C_{N_{p\alpha}}$, the first four are functions of Mach number. These functions have been tabulated and are used in the current code to calculate the meteorological quantities. This code performs straight-line interpolation for Mach values between two entries. The last two aerodynamic coefficients are assumed to be constant:

$$C_{D2} = 4.0 \qquad C_{N_{p\alpha}} = -0.75$$

The muzzle velocity for the M107 projectile was 397.4 m/s. The initial spin was taken to be 1148 rad/s.

Exactly the same aerodynamic behavior was assumed in the fitting equations as was used in the equations for generating the trajectories. For this caliber shell, it is generally conceded that the drag coefficient is known to within a half-percent. The significance of such an error will be explored in the simulations.

Preliminary Analysis

A set of uniquely valued solution parameters is desired that yields an absolute minimum of the sum of the squares for the residue values. Depending on the problem, the least squares fitting approach can yield other solutions with local larger minimums. The first consideration is whether the parameter solution of the flight dynamic equations is unique. The value A in Eq. (1) contains C_D and the parameter ρ as cofactors. The value of C_D depends on the temperature, which has a parameter to be determined. Therefore, the value A would seem to contain two factors that can vary and still leave the value of A unchanged. However, the value of T depends on ρ through a constraining equation, and uniqueness should be restored. Also, the parameter C_D is weakly influenced by other variables because C_D depends on the yaw of repose.

Another consideration is the sensitivity of the parameter values to errors in measurement. Consider the shell at apogee, where the shell velocity is aligned approximately along the X_1 direction. With these conditions, the MPM model yields much simplified expressions that are, in component form,

$$\dot{U}_1 = -AV_1 \tag{16}$$

$$\dot{U}_2 = G_2/(1 + h_a) \tag{17}$$

$$\dot{U}_3 = \frac{h_L G_2}{(1 + h_a)(1 - h_M)} \tag{18}$$

Assuming flat fire, a transformation is performed from the time to the X_1 coordinate with the focus on Eq. (16):

$$U_1' = -a(1 - W_1/U_1)(U_1 - W_1) \tag{19}$$

in which $a = A/(V/\ell)$. This equation can be further simplified with the assumptions of no winds. Transforming to nondimensionalized variables, $x = (X_1 - X_{1b})/\ell$ and $\bar{t} = U_{1b}(t - t_b)/\ell$, and integrating, it is obtained that

$$x = \frac{\ln(1 + a\bar{t})}{a} \tag{20}$$

For a small enough segment, Eq. (20) can be approximated by expanding in a series truncated to two terms:

$$x \approx \bar{t}(1 - a\bar{t}/2) \tag{21}$$

The question of the accuracy with which one can determine ρ can be examined by using Eq. (21). Radar and GPS measurement techniques smooth the output positions and velocities so that one is not aware of the fundamental error spread of the basic data. Given that this error in starting and ending positions is Δx, find the required distance x of the segment to make the relative density error smaller than a certain value, or more generally, the relative error in a. The logarithm of Eq. (21) is differentiated to obtain

$$\frac{\Delta x}{x} \approx -\frac{\bar{t}^2 \Delta a}{2(\bar{t} - a\bar{t}^2/2)} \tag{22}$$

The quantity in parentheses in the denominator on the right-hand side is approximately equal to x for sufficiently small $\bar{t}$. Perform the substitution and rearrange to obtain

$$x \approx \sqrt{|2\Delta x/a(\Delta a/a)|} \tag{23}$$

Here, $(\Delta a)/a$ is the relative uncertainty in a that can be tolerated and Δx is the uncertainty in x attributable to measuring system limitations.

The value of C_{D0} is generally not known to be better than a half-percent. This uncertainty is partly attributable to limitations of measuring equipment for the aerodynamics range and partly to round-to-round differences. Thus, the relative error of a cannot be less than a half-percent. The value of x is a minimum value that will give $\Delta a/a$ the allowed precision. A representative value of a for the M107 projectile at ground level for the numerical experiment is $a_0 = 1.3 \times 10^{-5}$. The drag decreases as the shell ascends and decelerates. Likewise, the density decreases with altitude and the value of a decreases accordingly. The resulting envelope for the minimum uncertainty for the trajectory length in which a relative uncertainty of a is $\leq 1\%$ is shown in Fig. 2. For the test trajectory at or near apogee, the value of a is only about a fifth of its value on the ground. In practice, the relative uncertainty in a will be on the order of the relative uncertainty in the quantity ρ, which is to be found.

From Fig. 2, it can be seen that the segment lengths for the M107 ($\ell = 0.155$ m) must be $x \geq 13{,}000$ ($13{,}000 \times 0.155 \approx 2000$ m) if the uncertainty in the segment length is 1 m ($\Delta x = 1/0.155$ m)

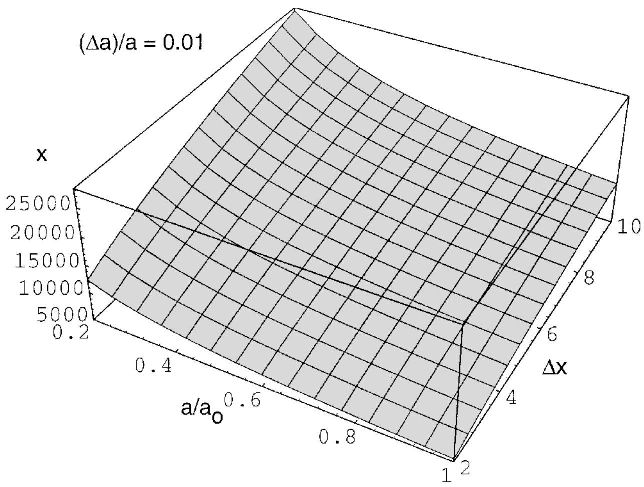

Fig. 2 Required segment length (caliber) to achieve given accuracy in *a* as a function of *a* and uncertainty in segment length.

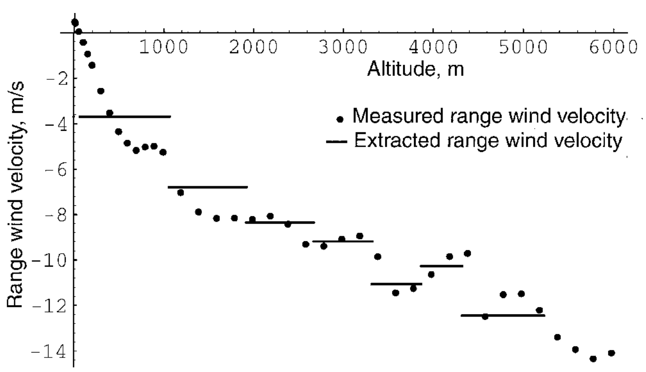

Fig. 3 Comparison of range wind velocity W_1 data with fitted values.

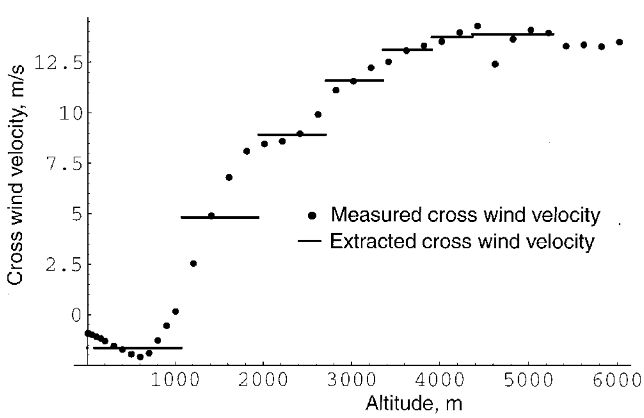

Fig. 4 Comparison of crosswind velocity W_3 data with fitted values.

near apogee, where the density times the drag coefficient is only a fifth of what it is at launch ($a/a_0 = 0.2$). The length of such a segment is large and might preclude obtaining sufficiently accurate meteorological estimates in the prescribed altitude zones.

Test Results and Discussion

The trajectory was fitted along its intervals in a sequential fashion, starting at ground level and using the ending values of the intervals as an initial estimate for the next interval. Although the initial estimates also serve as the unchanging initial values for the density and temperature, for the wind velocity values, initial estimates are adjusted to obtain the best fit along with the fitted lapse rate. The wind velocity data, together with the curves found by the least squares fitting method, are shown in Figs. 3 and 4.

The profiles are roughly reminiscent of boundary-layer behavior, as one would expect. The wind velocities are low near the ground and, at first, generally increase with altitude. The agreement of the least squares fitting results with the data is good, especially considering that a constant value of wind velocity is assumed for each segment. These fitted values, although not fitted on the same intervals as are used for firing tables, are in a format similar to the standard MET message.

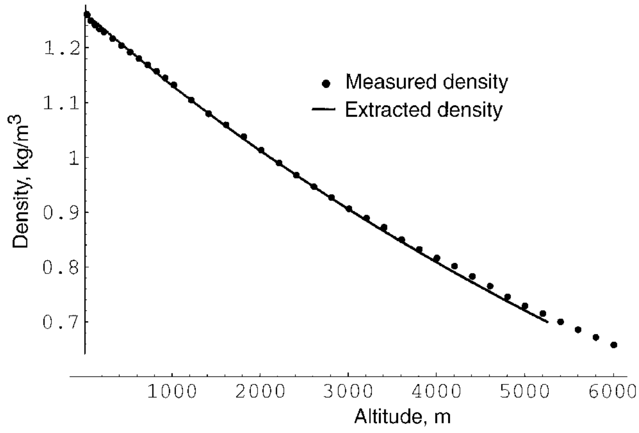

Fig. 5 Comparison of density data with fitted values.

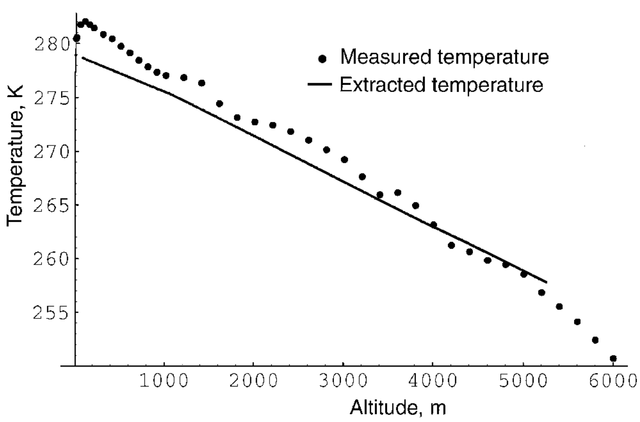

Fig. 6 Comparison of temperature data with fitted values.

The density and temperature data are compared with the fitted data in Figs. 5 and 6. The fitted density agrees best with the data near the ground and agrees less well near apogee because the calculation is started at the lowest lying segment and proceeds stepwise upward with the ending density value for the lower segment becoming the initial value for the higher segment. The temperature values obtained throughout the fitting calculations differ from the experimental temperatures by a few degrees over most of the trajectory. The initial value for the temperature is taken on the ground. A temperature inversion, which often occurs in the winter, results in a rapid initial increase in error with height. An error in the temperature chiefly affects the value of the zero yaw-of-repose drag coefficient C_{D0}, which depends weakly on the temperature for supersonic velocities, strongly for transonic velocities, and not at all for the low to moderate subsonic velocities. The first part of the trajectory occurs at supersonic and transonic velocities. The error in the temperature values also affects the density values because the change in the density depends on the change in the temperature.

Errors Induced by Measurement Errors and Model Assumptions

Figure 7 shows the positional errors in meters as a function of altitude. The positional error is the difference between the fitted value of the position and the generated value of the position. A large error in the altitude of 0.3 m is not shown here to allow closer examination of the other much smaller errors. The large errors at the lower altitudes occur where the projectile is traveling at supersonic and transonic speeds. In these regions, the value of the drag coefficient is a strong function of the temperature while being weakly dependent on the temperature in the subsonic region.

Table 1 Errors incurred from stale MET messages, M483A1 shell

Staleness, h	σ_{wind}, m/s	σ_{den}, %	σ_{temp}, %	σ_{ran}, m	σ_{defl}, m
0	0.4	0.15	0.25	14	8
1	2.1	0.40	0.30	66	40
2	2.5	0.69	0.57	84	49
4	3.7	0.97	0.79	122	71
∞	5.7	6.60	3.0	341	109

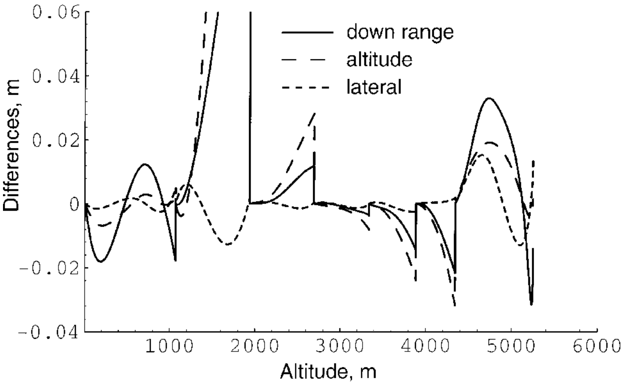

Fig. 7 Error in fitted position (meters) vs altitude.

As discussed earlier, there will be an uncertainty in the measurements of the end positions of a trajectory segment that will induce errors in obtaining the fitted values of the meteorological quantities. Uncertainty in measured positions along the trajectory segment should also induce errors in fitting the atmospheric values. Another source of uncertainty is the actual value of the drag coefficient C_D. The value of the drag coefficient is commonly known to within only a half-percent, as discussed earlier. Additional error may be incurred from lot-to-lot variations. To assess the level of uncertainty in the meteorological parameters caused by uncertainty in the value of the drag coefficient, the fitted value of the drag coefficient was assumed to be a half-percent lower than the drag coefficient values used in generating the trajectory. The resulting fitted density was approximately a half-percent higher than had been obtained previously.

The consequences of using normally stale density and temperature data to obtain equivalent winds can also be briefly examined. Table 1 shows the standard deviation of atmospheric quantities measured at selected times after an initial measurement.[1] Shown also are the associated error magnitudes for range and deflection for the M483A1 shell fired from the M109A2 howitzer. The target range is 15,000 m with the zone 8 charge. Table 1 shows that fresh atmospheric data yield nonzero targeting error values that are caused by instrument-measuring errors. The errors incurred by ignoring atmospheric conditions are large, as shown in the last row. If it is assumed that the density and temperature could be approximated by a standard atmospheric model, then using a shell's measured trajectory to find the wind velocity by a fitting process could yield large fictitious wind velocities.

Conclusions

By using a least squares fitting procedure, atmospheric conditions were obtained knowing only the projectile's trajectory and its aerodynamic coefficients. The projectile trajectory was generated using the MPM model, and the atmospheric conditions were supplied by a weather balloon. A least squares fit of several trajectory segments that comprised altitude zones was made with the assumption of constant wind speeds in each segment. The fitting procedure started from the ground and used the fitted density and temperature conditions at the endpoint of the trajectory segment as the initial fitted conditions for the next segment. The agreement of the fitted meteorological values with the balloon data was good, but the agreement decreased with the increasing amounts of noise added to the trajectory positions. To obtain atmospheric conditions with the needed precision, the accuracy analysis shows that the starting and ending positions of segment lengths must be known with more precision than current measuring techniques are capable of providing.

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